Light-front projections of the Bethe-Salpeter amplitude and the 4D electromagnetic current for an interacting two-fermion system

T Frederico[1], J A O Marinho [1,2], E Pace[3] and G Salmè[4]

[1] Dep. de Física, Instituto Tecnológico de Aeronáutica, 12.228-900 São José dos Campos, São Paulo, Brazil

[2]Centro de Física Computacional, Department of Physics, University of Coimbra, Portugal

[3] Dipartimento di Fisica, Università di Roma "Tor Vergata" and Istituto Nazionale di Fisica Nucleare, Sezione Tor Vergata, Via della Ricerca Scientifica 1, I-00133 Roma, Italy

[4]Istituto Nazionale di Fisica Nucleare, Sezione Roma I, P.le A. Moro 2, I-00185 Roma, Italy

E-mail: salmeg@roma1.infn.it

Abstract. A recent approach for constructing an exact 3D reduction of the 4D matrix elements of the electromagnetic current for an interacting two-fermion system, is briefly reviewed. The properties of the obtained 3D current, like the fulfillment of the Ward-Takashi Identity and its Fock decomposition, will be illustrated in relation with future applications to few-nucleon systems.

1. Introduction

A detailed investigation of the electromagnetic (em) properties of hadrons and nuclei requests a careful treatment of the current operator for interacting fermion systems. In particular, besides general properties like the Poincaré covariance, the gauge symmetry of the em interaction plays the well-known essential role, that leads to the Ward-Takahashi Identity (WTI) for the em vertex function. In this short presentation, some insights will be given on the main ingredients of our approach [1] aimed to construct a 3D em current for an interacting two-fermion system, that i) fulfills WTI and therefore allows the current conservation (CC) and ii) preserves at large extent the Poincaré covariance, emphasizing the relevant steps for obtaining workable expressions to be used in actual calculations.

Field theoretical approaches, based on the Bethe-Salpeter equation, (see, e.g. [2, 3, 4]) allow one to evaluate matrix elements of the 4D em current for an interacting two-body system, viz

$$\langle P_f, J_f, \sigma_f | j^\mu | P_i, J_i, \sigma_i \rangle = \mathcal{C} \sum_{\tau_i,..} \int \frac{d^4k_1}{(2\pi)^4} \bar{\Psi}^{J_f}_{\sigma_f}(k_1, k_2', P_f) \mathcal{J}^\mu(k_1, k_2', k_2) \Psi^{J_i}_{\sigma_i}(k_1, k_2, P_i) \quad (1)$$

where, in the present case, $\Psi^{J\sigma}(k_1, k_2, P)$ is the Bethe-Salpeter (BS) amplitude of a two-fermion systems (with $k_1 + k_2' = P_f$ and $k_1 + k_2 = P_i$ for the final and initial state, respectively), $\mathcal{C}$ contains the normalization factors, and $\mathcal{J}^\mu$ is the 4D fermion-photon vertex (or kernel) that contains one + two +...+ many-body contributions, and must fulfill WTI. The sum in Eq. (1) is performed on isospin, spinorial, etc. indexes.

To solve the BS equation in Minkowski space for obtaining $\Psi^{J\sigma}(k_1, k_2, P)$, in realistic cases, is still a prohibitive task, but new and very effective approaches are coming into play (see, e.g. [5]). On the other hand, 3D reductions are well known tools (see, e.g. [6]) for attempting to overcome the above mentioned difficulties, but still saving at large extent the general properties that the em current has to fulfill.

Aim of our investigation [1] is i) to achieve an exact 3D reduction of the 4D matrix elements in Eq. (1), i.e. without loosing any physics content, and ii) to elaborate a systematic analysis of the obtained expression in order to develop effective approximation. The main ingredients for our approach, that were already applied to the case of two-boson interacting system in [4], are in order i) the Quasi-Potential (QP) approach for the Transition matrix (see [7]), and ii) the projection of the 4D physical quantities onto the 3D Light-front (LF) hyperplane (i.e. $x^+ = x^0 + x^3 = 0$) (see, e.g. [2, 3]). As a first result, one can establish a formally exact correspondence between 4D BS amplitude and the 3D LF "valence" wave function. Then it is possible to express, without approximation, the matrix elements of the 4D em current, where the BS amplitudes appear, in terms of matrix elements of a 3D LF current, that fulfills the WTI. It should be pointed out that in these last matrix elements the valence states are present.

Within the Quasi-Potential approach for the T-matrix, where the so-called *effective interaction* appears, one can develop a meaningful approximation scheme for evaluating the 3D matrix elements, based on the Fock decomposition of the effective interaction. In particular, one expects a decreasing influence of the Fock states that contain an increasing number of intermediate particles. This machinery allows one to construct an iterated approximation of the effective interaction and eventually to evaluate matrix elements of the "truncated" LF em current, that still fulfills at any order the LF WTI. Finally, by using a Yukawa model in ladder approximation, as a pedagogical example, explicit expressions for many-body contributions to the em LF current have been obtained [1].

2. BS amplitudes and EM current

The quantities present in Eq. (1) are the BS amplitude and the em vertex. In particular, the BS amplitude $|\Psi\rangle$, that depends upon *internal variables*, satisfies the following equation (the $\lim_{\varepsilon\to 0}$ is understood)

$$G^{-1}(K)\,|\Psi\rangle = 0 \tag{2}$$

with appropriate boundary conditions for bound and scattering states. In Eq. (2), K is the 4-momentum of the center of mass and the inverse of the Green function is given by

$$G^{-1}(K) = G_0^{-1}(K) - V(K) \tag{3}$$

where $V(K)$ is the 4D interaction, as obtained by the Lagrangian, $G_0(K) = S(k_1)\; S(K - k_1)$, with $S(k)$ the Dirac propagator.

The em vertex, $\mathcal{J}^\mu$, corresponding to the interaction $V(K)$ has a free term, $\mathcal{J}_0^\mu$, and another one, $\mathcal{J}_I^\mu$, which depends on the interaction, as requested by the commutation rules with the generators of the Poincarè group. It also satisfies the following Ward-Takahashi identity

$$Q_\mu \mathcal{J}^\mu(Q) = G^{-1}(K_f)\widehat{e}\; - \widehat{e}\; G^{-1}(K_i) \tag{4}$$

where Q^μ is the 4-momentum transfer and $\widehat{e} = \widehat{e}_1 + \widehat{e}_2$ is the pointlike charge operator with matrix elements given by $\langle k_j|\widehat{e}_j|p_j\rangle = e_j \delta^4\left(k_j - p_j - Q\right)$

Combining Eqs. (3) and (4) with the decomposition of $\mathcal{J}^\mu$ in terms of the dependence upon $V(K)$, it turns out that each contribution fulfills a suitable constraint, viz

$$Q_\mu \mathcal{J}_0^\mu(Q) = G_0^{-1}(K_f)\widehat{e}\; - \widehat{e}\; G_0^{-1}(K_i)$$

and

$$Q_\mu \mathcal{J}^\mu_I(Q) = \widehat{e}\, V(K_i) - V(K_f)\widehat{e}\ .$$

Finally, CC can be explicitly retrieved by using solutions of the BS equation, Eq. (2), i.e.,

$$Q_\mu \langle \Psi_f | \mathcal{J}^\mu(Q) | \Psi_i \rangle = \langle \Psi_f | \left[G^{-1}(K_f)\widehat{e}\ - \widehat{e}\, G^{-1}(K_i) \right] | \Psi_i \rangle = 0 \quad . \tag{5}$$

After introducing the general equations fulfilled by BS amplitudes and em current, let us illustrate our procedure. As well known, the T-matrix is given in terms of the interaction $V(K)$ by an integral equation: $T(K) = V(K) + V(K)\ G_0(K)\ T(K)$.

In the Quasi-Potential framework [7], $T(K)$ can be rewritten in terms of two coupled equations

$$T(K) = W(K) + W(K)\ G_{aux}(K)\ T(K) \tag{6}$$

where the effective interaction $W(K)$, in turn, is a solution of

$$W(K) = V(K) + V(K)\ \Delta_0(K)\ W(K) \tag{7}$$

with $\Delta_0(K) := G_0(K) - G_{aux}(K)$, and $G_{aux}(K)$ is a smart choice to speed up the convergence of the iterative solutions.

3. Projecting onto the Light-Front hyperplane

In Eq. (1) the 4D integration contains all the difficulty given by the very complicated analytic structure of both BS amplitudes and em vertex in Minkowski space. The adopted strategy for decreasing the degree of difficulty, is based on disentangling the "dynamical" variable, i.e. the time, from the remaining three kinematical variables, that describe the "initial" state. In order to accomplish such a task, the LF kinematics appears a very appealing tool (see, e.g. [8] for an extensive review) since the LF combination of the Poincaré generators has the property to produce the maximal numbers of generators without interaction (7 out of 10). In turn, such a property produces many other relevant features, like the kinematical nature of the LF boosts (to be not confused with the standard boosts, a part the longitudinal one). In particular, the LF components of the 4-momentum and the scalar product are given by

$$k^\pm = k^0 \pm k_z, \qquad \mathbf{k}_\perp \equiv \{k_x, k_y\}, \qquad k\cdot x = \frac{(x^- k^+ + x^+ k^-)}{2} - \mathbf{x}_\perp \cdot \mathbf{k}_\perp \quad . \tag{8}$$

Integrating a given physical quantity, $\phi(k)$, over the minus component of the 4-momentum, k^-, amounts to restrict its dependence upon only 3 variables in the coordinate space. One has

$$\begin{aligned}\int dk^-\ \phi(k) &= \int dk^-\ \frac{1}{2(2\pi)^4} \int dx^+\ dx^-\ d\mathbf{x}_\perp\ e^{ik\cdot x}\ \tilde{\phi}(x) = \\ &= \frac{1}{(2\pi)^3} \int dx^+\ dx^-\ \delta(x^+) d\mathbf{x}_\perp\ e^{i(x^- k^+/2 - \mathbf{x}_\perp \cdot \mathbf{k}_\perp)}\ \tilde{\phi}(x) \quad .\end{aligned} \tag{9}$$

Therefore, the "LF-time", x^+, that labels the dynamical evolution (as the standard "t" in the instant form of the relativistic Hamiltonian), is constrained to its initial value, i.e. $x^+ = 0$. Therefore, once the physical quantities are integrated over k^-, they are constrained to live onto the LF hyperplane defined by $x^+ = 0$.

A crucial step for the LF projection is the separation of the fermion propagator in an on-shell term and in an instantaneous one, i.e

$$i\ S(k) = \frac{\not{k} + m}{k^2 - m^2 + i\varepsilon} = \frac{\not{k}_{on} + m}{k^+(k^- - k^-_{on} + \frac{i\varepsilon}{k^+})} + \frac{\gamma^+}{2k^+} = i\ S_{on}(k) + \frac{\gamma^+}{2k^+} \tag{10}$$

where $k^-_{on} = (\mathbf{k}^2_\perp + m^2)/k^+$ is the minus-component of k^μ_{on}, such that $k_{on} \cdot k_{on} = m^2$, and the second term leads to an instantaneous (in LF time!) free propagation, since the formal Fourier transform generates $\delta(x^+)$ (see Eq. (9)). It should be pointed out that such a term makes the treatment of a fermionic system basically different from the treatment of a bosonic one. On the other hand, the decomposition in Eq. (10) suggests a strategy for achieving our goal of an exact 3D reduction. As a matter of fact, let us define the 4D on-shell two-body Green's function [1, 4]

$$G^{on}_0(K) := S_{on}(k_1)\ S_{on}(K - k_1) \tag{11}$$

that plays a key role in our approach, since allows us to separate the *trivial* 4D propagation associated to such a contribution.

The 3D LF counterpart of $G^{on}_0(K)$ is introduced by integrating over k^-_1 and k'^-_1, i.e. by projecting onto the 3D LF hyperplane, the following matrix elements (note that the other three components of the momentum remain operators)

$$g_0(K) = \ \downarrow G^{on}_0(K) \downarrow\ \equiv \Omega^+ \int dk'^-_1 dk^-_1 \left\langle k'^-_1 | G^{on}_0(K) | k^-_1 \right\rangle \tag{12}$$

where Ω^+ is a suitable kinematical factor ensuring the kinematical Poincaré covariance (e.g. $\Omega^+ = k^+$ or $\Omega^+ = \sqrt{k^+(K^+ - k^+)}$), and the symbol $\downarrow$, from now on, indicates the integration over the minus component of the 4-momentum that appears on the left or on the right side, depending upon the position of the symbol itself.

In the case without interaction, it is easy to find the relation between the 3D LF free state, $|\phi_0\rangle$, solution of $g^{-1}_0(K)\ |\phi_0\rangle = 0$ (note that instantaneous terms do not contribute) and the free BS amplitude, solution of $G^{-1}_0(K)\ |\Psi_0\rangle = 0$ (note that the particles are on their own mass-shell in the free case). It turns out [1] that

$$|\Psi_0\rangle = G^{on}_0(K)\!\downarrow g^{-1}_0(K)\ |\phi_0\rangle = \Pi_0(K)\ |\phi_0\rangle \tag{13}$$

where $\Pi_0(K)$ is the *free reverse LF projector*, that allows to jump from a 3D description to a 4D world. The operator $\Pi_0(K)$ has a simple explicit expression [1, 4] in terms of the positive energy Dirac projector. Let us remind that, by construction $\downarrow\!|\Psi_0\rangle = |\phi_0\rangle$: this completes the reconstruction pattern in the simple case of a free two-fermion system.

In order to apply the projection technique to the QP approach, namely for an interacting case, one needs an intermediate step, given by a suitable choice for the auxiliary Green's function to be used in the 4D operator Δ_0, see Eq. (7). The chosen 4D auxiliary Green's function is defined as follows in terms of the 3D $g_0(K)$

$$G_{aux}(k) = \Pi_0(K) g_0(K) \overline{\Pi}_0(K) \tag{14}$$

where $\overline{\Pi}_0(K) = g^{-1}_0(K) \downarrow\! G^{on}_0(K)$. Then, the T-matrix and the effective interaction $W(K)$ can be rewritten as follows

$$\begin{aligned} T(K) &= W(K) + W(K)\ \left[\Pi_0(K) g_0(K) \overline{\Pi}_0(K)\right]\ T(K) \\ W(K) &= V(K) + V(K)\ \Delta_0(K)\ W(K) = \\ &= V(K) + V(K)\ \left[G_0(K) - \left(\Pi_0(K) g_0(K) \overline{\Pi}_0(K)\right)\right]\ W(K) \quad . \end{aligned} \tag{15}$$

The equation for $W(K)$ can be solved by iteration, i.e.

$$W(K) = V(K)\ \sum_{i=1}^{\infty} [\Delta_0(K) V(K)]^{i-1} \quad . \tag{16}$$

The rate of convergence of this series is related to $\Delta_0(K)$, that describes the difference between the intermediate propagation of two non-interacting fermions and the 4D counterpart of the LF propagator $g_0(K)$. Depending upon the powers of $\Delta_0(K)$, we will have intermediate states with more and more bosons and fermions (once we introduce time-ordered diagrams).

The Rosetta Stone relating 4D and 3D quantities, is readily constructed. The 3D T-matrix and effective interaction are respectively

$$t(K) = \overline{\Pi}_0(K)\ T(K)\ \Pi_0(K) \qquad\qquad w(K) = \overline{\Pi}_0(K)\ W(K)\ \Pi_0(K) \quad . \tag{17}$$

Once $t(K)$ is reexpressed in terms of $g_0(K)$ and $w(K)$, one can introduce the 3D interacting Green's function and its inverse as follows

$$g(K) = g_0(K) + g_0(K)t(K)g_0(K) \qquad\qquad g^{-1}(K) = \overline{\Pi}_0(K)\ G^{-1}(K)\ \Pi(K) \tag{18}$$

where the *interacting LF reverse projection operator* is

$$\Pi(K) = \ [1 + \Delta_0(K)\ W(K)\ ]\ \Pi_0(K) \quad . \tag{19}$$

The solution $|\phi\rangle$ of the following 3D equation,

$$g^{-1}(K)\,|\phi\rangle = \left[g_0^{-1}(K) - w(K)\right]\ |\phi\rangle = 0 \tag{20}$$

with appropriate boundary conditions for bound and scattering states, is the valence component of the LF wave function for two interacting fermions, as can be deduced by the relation with the BS amplitude, shown in what follows. The exact relations between the 3D valence component and the 4D BS amplitude can be readily obtained, and they are given by

$$|\Psi\rangle = \Pi(K)\ \ |\phi\rangle \qquad\qquad \downarrow G_0^{on}(K)\ G_0^{-1}(K)\ \ |\Psi\rangle = \ \ |\phi\rangle \quad . \tag{21}$$

The rightmost equation illustrates the motivation for calling $|\phi\rangle$ valence component, given the presence of the integration on the minus component in lhs (see, e.g [5] and references quoted therein). Such an integration makes only the first component of the Fock decomposition of $|\Psi\rangle$ to survive.

A possible approximation scheme is suggested by the QP expansion of the effective interaction, Eq. (16). Then, $w(K)$, $t(K)$ and the 3D LF Green's function $g(K)$ (all containing $W(K)$ in a more or less explicit way) can be evaluated by truncating the expansion at a given order. It should be pointed out that the full complexity of the Fock-space affects $g(K)$ through the effective interaction $w(K)$, as can be seen through a time-ordered analysis of the diagrams associated to each contribution $[\Delta_0(K)V(K)]^{i-1}$, and reminding that $\Delta_0(K)$ describes propagation. Indeed, the truncation of the QP expansion limits the number of Fock states involved in the construction of the effective interaction. As a final remark, one could argue that the convergence rate of the QP expansion is related to the small probability of the higher Fock-components.

4. The 3D EM current and the LF Ward-Takahashi Identity

Once we have obtained the relation between the 4D BS amplitude and the 3D LF valence wave function, one can establish a direct link between the matrix element of the 4D current and the matrix elements of the 3D LF em current operator.

For both scattering and bound states one has [1, 4, 2, 3]

$$\langle\Psi_f|\ \mathcal{J}^\mu(Q)\,|\Psi_i\rangle = \langle\phi_f|j^\mu(K_f,K_i)|\phi_i\rangle \tag{22}$$

where the 3D LF current operator, acting on the valence wave functions, is defined as follows

$$j^{\mu}(K_f, K_i) = \overline{\Pi}(K_f)\mathcal{J}^{\mu}(Q)\Pi(K_i) = \\ = \overline{\Pi}_0(K_f)\left[1 + W(K_f)\Delta_0(K_f)\right]\mathcal{J}^{\mu}(Q)\left[1 + \Delta_0(K_i)W(K_i)\right]\Pi_0(K_i) \quad . \tag{23}$$

Since the 4D current must fulfill the WTI, Eq. (4), the 3D LF current fulfills the following LF WTI [1]

$$Q_{\mu}j^{\mu}(K_f, K_i) = \overline{\Pi}(K_f)\left[G^{-1}(K_f)\hat{e} - \hat{e}G^{-1}(K_i)\right]\Pi(K_i) = \\ = g^{-1}(K_f)\ \hat{\mathcal{Q}}^{L}_{LF} - \hat{\mathcal{Q}}^{R}_{LF}\ g^{-1}(K_i) \tag{24}$$

where $\mathcal{Q}^{L(R)}_{LF}$ is called the left (right) LF charge operator (note that it is *interaction free !*). For the sake of concreteness, let us show the explicit expression of the left charge, viz

$$\hat{\mathcal{Q}}^{L}_{1LF} = \Lambda_{+}(\hat{k}_{1on})\frac{m_1}{\hat{k}^{+}_{1}}\gamma^{+}_{1}\ \hat{e}_{1LF}\Lambda_{+}(\hat{k}_{1on})\Lambda_{+}(\hat{k}_{2on}) \tag{25}$$

where Λ_+ is the positive energy Dirac projector, and $\hat{e}_{1LF}$ the 3D pointlike charge operator, given by

$$\langle k'^{+}_{1}, \vec{k}'_{1\perp}|\hat{e}_{1LF}|k^{+}_{1}, \vec{k}_{1j\perp}\rangle := e_1\delta\left(k'^{+}_{1} - k^{+}_{1} - Q^{+}\right)\ \delta^2\left(\vec{k}'_{1\perp} - \vec{k}_{1\perp} - \vec{Q}_{\perp}\right) \quad . \tag{26}$$

The current conservation straightforwardly follows by taking the matrix elements of Eq. (24) between 3D interacting states $|\phi_{i(f)}\rangle$, solutions of $g^{-1}(K)\ |\phi\rangle = 0$

For the actual calculations, one needs to truncate the iterated expression of the effective interaction $W(K)$, Eq. (16), and therefore one ends up with a truncated 3D em current. The truncation scheme for the LF current cannot be carried out in a naive way, if one has to fulfill a WTI. Indeed, at each truncation order it will correspond a truncated effective interaction, $w^{(n)}(K)$, a truncated LF Green's function $g^{(n)}(K)$, a truncated valence wave function, $|\phi^{(n)}\rangle$, a properly truncated 3D LF current, and finally a truncated WTI. Let us see in details this chain.

Once we truncate the QP expansion of the effective interaction at the order n,

$$W^{(n)}(K) = V(K)\sum_{i=1}^{n}\left[\Delta_0(K)V(K)\right]^{i-1} \tag{27}$$

we can immediately generate an approximate 3D LF Green's function, $g^{(n)}(K)$, leading to an approximate valence wave functions, $|\phi^{(n)}\rangle$, solutions of the following equation

$$[g^{(n)}(K)]^{-1}\ |\phi^{(n)}\rangle = \left[g^{-1}_{0}(K) - w^{(n)}(K)\right]\ |\phi^{(n)}\rangle = 0 \quad . \tag{28}$$

The corresponding em current, that fulfills CC, if one uses $|\phi^{(n)}\rangle$ for evaluating the matrix elements, cannot be directly constructed inserting in the definition of the 3D current, Eq. (23), the truncated $W^{(n)}(K)$, i.e.

$$Q\cdot j^{(n)}(K_f, K_i) \neq [g^{(n)}(K_f)]^{-1}\ \hat{\mathcal{Q}}^{L}_{LF} - \hat{\mathcal{Q}}^{R}_{LF}\ [g^{(n)}(K_i)]^{-1} \tag{29}$$

with

$$j^{(n)\mu}(K_f, K_i) = \overline{\Pi}_0(K_f)\left[1 + W^{(n)}(K_f)\Delta_0(K_f)\right]\mathcal{J}^{\mu}(Q)\ \times \\ \left[1 + \Delta_0(K_i)W^{(n)}(K_i)\right]\Pi_0(K_i) \quad . \tag{30}$$

The fulfillment of the LF WTI excludes such a naive approximation scheme for the truncated current, but, at the same time, gives us the correct hint. One should implement a power counting of the effective interaction that appears in the definition of the LF current (the interaction $V(K)$ is present both in the projectors $\Pi(K)$ and in the 4D current $\mathcal{J}$) and in the rhs of the LF WTI, where only $[g^{(n)}(K_f)]^{-1}$ and $[g^{(n)}(K_i)]^{-1}$ are affected by the interaction, as in the exact case, i.e. Eq. (24).

For the truncation at the n-th order, a truncated current that satisfies a truncated LF WTI is given by [1]

$$j^{c(n)\mu} = j^{c(n-1)\mu} + \overline{\Pi}_0 \left[\sum_{i=0}^{n} W_i \Delta_0 \mathcal{J}_0^\mu \Delta_0 W_{n-i} + \sum_{i=0}^{n-1} W_i \Delta_0 \mathcal{J}_I^\mu \Delta_0 W_{n-1-i} \right] \Pi_0 \tag{31}$$

where $W_i = V\,[\Delta_0 V]^{i-1}$ and it has been formally defined $W_0 \Delta_0 = \Delta_0 W_0 = 1$. It should be reminded that $\mathcal{J}_0^\mu$ is $O(V^0)$ and $\mathcal{J}_I^\mu$ is $O(V^1)$.

Then $j^{c(n)\mu}$ satisfies a LF WTI given by

$$Q^\mu j_\mu^{c(n)} = [g^{(n)}]^{-1}(K_f)\ \widehat{\mathcal{Q}}_{LF}^L - \widehat{\mathcal{Q}}_{LF}^R\ [g^{(n)}]^{-1}(K_i) \quad . \tag{32}$$

For obtaining CC, the matrix elements should be taken between solutions of $[g^{(n)}]^{-1}|\phi_n\rangle = 0$.

5. Two-body current in the LF approach: a pedagogical example

A Yukawa model in ladder approximation has been adopted in order to elaborate an application of our general approach. Within such a model, a couple of fermions interacts by the exchange of a boson, in the present case chargeless, for simplicity. From the Lagrangian, one can deduce immediately the 4D interaction to be used in the definition of the T-matrix and of the 4D current. In ladder approximation, the 4D current is nothing else but the free 4D current that fulfills the suitable WTI (see, e.g. [9]). Let us stress that, though in Minkowski space the current of the model under investigation is the free one, its projection onto a 3D hyperplane becomes interaction dependent. The matrix elements of the first-order LF current, in a 3D Fourier space, are given by

$$\langle k_1'^+ \vec{k}_{1\perp}' | j^{c(1)\mu} | k_1^+ \vec{k}_{1\perp} \rangle = \langle k_1'^+ \vec{k}_{1\perp}' | \left[j^{c(0)\mu} + \overline{\Pi}_0 \left[V\ \Delta_0 \mathcal{J}_0^\mu + \mathcal{J}_0^\mu \Delta_0\ V \right] \Pi_0 \right] | k_1^+ \vec{k}_{1\perp} \rangle \tag{33}$$

with $j^{c(0)\mu} = \overline{\Pi}_0 \mathcal{J}_0^\mu \Pi_0$ (remind that Π_0 is the free-case projector). In figure 1, the diagrammatic analysis of the first-order LF current is shown. It should be pointed out that i) the diagrams are LF-time ordered, ii) the LF-time flows from the right to the left and iii) the external legs are particles on their-own mass shell (therefore only three components can independently change). Such a diagrammatic analysis allows one to appreciate the physical content inside the operator $j_\mu^{c(1)}$. The peculiar feature of the LF approach is the presence of instantaneous contributions, generated by the term proportional to γ^+ in Eq. (10). This contributions are labeled by horizontal dashes. Another interesting contribution is the Z-diagram, or pair production diagram, generated by the time-ordering of the 4D Dirac propagator. In this respect, we should emphasize that the full Dirac structure, i.e. the spinorial structure, is exactly taken into account, since the LF projection only works on the minus components of the four-momenta.

This first-order current will be applied for evaluating the Deuteron electromagnetic properties, in order to extend the analysis at the zero-order already performed [10]. But, one can anticipate that i) the pair term affects all the three electromagnetic form factors, while the instantaneous terms contribute only to the magnetic one (since the property $\gamma^+\ \gamma^+ = 0$ is acting in the charge form factor), ii) the pair term vanishes for $q^+ \to 0$, for the conservation law of the plus components, while the instantaneous ones survive, iii) the pair term should be maximal at $q^+ \sim m_N$ [11], iv) the remaining terms affect all the Deuteron form factors, in the whole range of q^+.

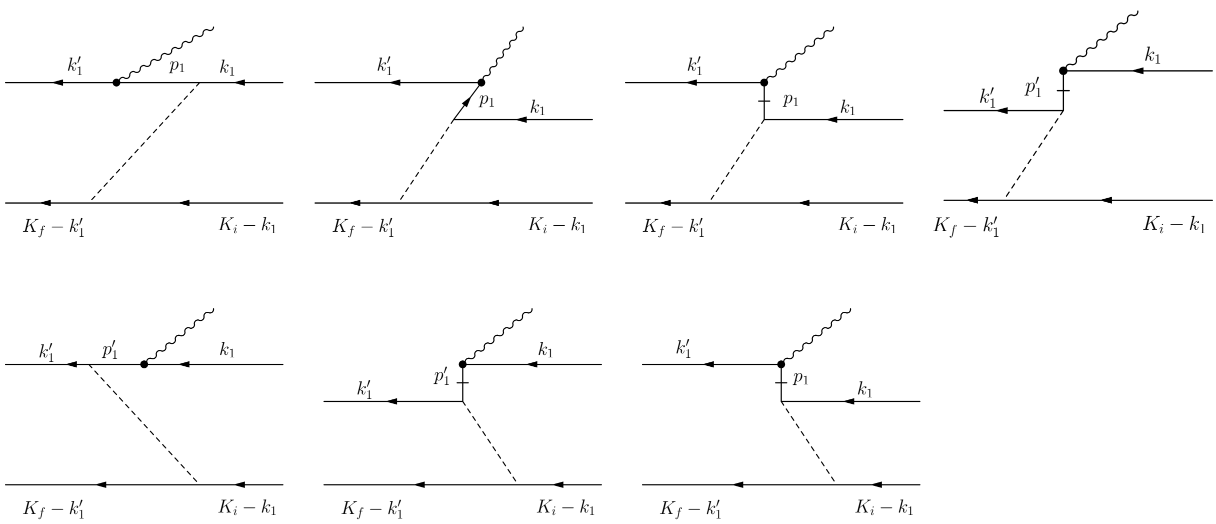

Figure 1. LF-time ordered diagrams for the first-order LF current for a Yukawa model in the ladder approximation, see Eq. (33) and text. Horizontal dashes indicate instantaneous propagations. (Adapted from [1])

6. Conclusions

For two interacting fermions, an exact correspondence between 4D BS amplitudes and 3D LF valence wave functions has been obtained [1], by using the projection onto the LF hyperplane. The approach fulfills the covariance with respect to the kinematical Poincaré subgroup (7 generators out of 10).

The obtained relation allows one to express the matrix elements of the 4D em current in terms of matrix elements of the 3D LF em current between valence wave functions. For such a current operator, the corresponding LF WTI can be constructed by introducing interaction free LF charge operators, then current conservation can be trivially retrieved.

Finally, by using a suitable auxiliary Green's function, within the Quasi-Potential approach, that allows an ordering in terms of intermediate Fock states, an approximation scheme has been developed, for both dynamical equation and the em current operator. In particular, at any order of the effective interaction the em current operator fulfills a proper LF WTI, leading to CC.

A systematic analysis of LF two-body currents, obtained within a Yukawa model in ladder approximation, is in progress for the Deuteron case.

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